How to Identify Basic Knowledge and Basic Skills?

Features of Modern General Education in Mathematics.

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1. Should Students be able to .....

Should students be able to

- *solve the equation* $x^2 + 4x + 4$ *?*
- *determine the maximum value of the function f when* $f(x) = x\,(1-x)$ *?*
- *calculate the integral* $\int_0^1 (x - x^2)\,dx$ *?*

Our answer is: of course, they should!

Should students be able to solve these problems without using CAS?
We have no answer to that question, rather we must pose a new question:
There are various possibilities for solving the problems mentioned above. Should it be forbidden for the students to use any number of other intelligent solution possibilities aside from CAS? If so, why? If not, why CAS?

Should we spend our teaching time extensively explaining to the students how to work out the solutions of such similar problems by hand?
Our answer is: That depends on the image of mathematics, the teaching thereof and mathematics general education which we ourselves have and it depends upon that image of mathematics which we want to convey to our students.

We ourselves are convinced that a mathematics classroom, in which the necessity of manually working out solutions and practicing them has taken the upper hand and has become so time consuming can hardly be rescued - and by that we mean the concrete teaching of mathematics as well as mathematics as a subject in general.

In order to have a rational discussion about a subject it is not sufficient just to have an opinion – one needs arguments! And furthermore, in order to have a scientific discussion one needs a theoretical frame of reference having found acceptance in the scientific community, allowing for classification and for a rational assessment of the arguments. The simpler this theoretical frame of reference is, the greater the chance of its acceptance even on the fringes and outside of the scientific community.

In the discussion that follows we would like to sketch a view of mathematics and of general mathematics education based on work done by our colleague, R. Fischer, in Klagenfurt and which is, in our opinion well suited to assess the experience and considerations of using CAS. Some considerations and examples illustrated here can also be found in (Schneider 1999, Schneider 2000).

2. Mathematics and General Mathematics Education

2.1 Mathematics as an Interplay between Representing, Operating and Interpreting

Current conception of mathematics is, for the most part, closely coupled to those of calculations. Calculation means the reshaping of mathematic facts according to rigidly formulated rules. Depending upon the level of mathematics, it is comprised of arithmetic combination of numbers, elementary algebraic operations with variables and terms, matrix operations, procedures for solving equations, inequations and of systems of equations and inequations, derivations, calculating integrals etc. up to developing deductive conclusions (proofs).
R. Fischer and G. Malle designate this „*generalized calculation*“ as "*operating by rules*“ (Fischer/Malle 1985, p. 221).

Operating by rules presumes that the *representation* of the fact occurs in such a way that operating by rules is made possible at all. (At a later time we will postulate this seemingly trivial fact as a constitutive feature of mathematics.) And: operating by rules only makes sense where the *interpretation* of the results of the reshaping provide relevant information in the context of the original fact which was not available before having carried out the operation.
One can view this interplay of representing, operating by rules and interpreting in many exercises outside of or within mathematics: a fact is *represented* by an arithmetic expression, as an equation or a system of equations and inequations, as an integral or as a differential equation. *Operating by rules* (carrying out arithmetic operations, solving equations, systems of equations or inequations, determining a derivative, calculating an integral and so on) leads to a new representation of the fact. The *interpretation* of this new representation delivers new information about the given context (for example: the amount of a bill, an optimal production program, local extremes, the content of the area underneath a curve).
These considerations lead R. Fischer to the following „*definition of mathematics*“ (Fischer 1990, p. 38), as simple as it is encompassing:

Mathematics = representing + operating + interpreting

Whereby the question immediately arises as to *what* is actually being represented in mathematics and *how* it is being represented in mathematics.

Normally, in mathematics we have *abstract* (not directly perceptible to the senses) *relationships* (a digit as a quantitative relationship, an equation as a relationship between variable sizes, a function as a relationship between the elements of two amounts and so on), which are *materialized* – primarily by means of written symbols, and in most recent years by pocket calculators, computers and the corresponding software –, that is they are represented by objects which are perceptible to the senses (cf. for example Fischer/Malle 1985, p. 221ff or Fischer 1990, p. 39f).

Of course, abstract relationships can also be materialized by other means such as through the written word. There is nothing special about the fact that in mathematical representations abstract relationships are materialized. More importantly, the significance lies in the fact that – as has oftentimes been cited and according to the above mentioned definition – these representations allow for regular reshaping (cf. Fischer 1999, p. 98f).

R. Fischer summarizes these thoughts in a further *„definition of mathematics"* (Fischer 1999, p. 90):

„Mathematics is the material, symbolic representation of abstract facts which cannot be directly perceived by the senses offering the possibility of regularized reshaping."

This „definition" refers to an obligation of mathematics to recognize (or invent) a representation for an abstract fact which allows for operating by rules; interpretation then means the explanation by means of facts represented in symbols (or also in another way).

In addition to regularized reshaping of symbolic representations, we must also „translate" the various (cognitive and material) representations. *„Doing mathematics consists of much more than the greater part of an interaction between the person and the form of representation (whether on the paper or on the screen). One alters the form of representation, contemplates it, alters it again, contemplates it, and so on and so on."* (Fischer o. J. a, p. 7).

The core statement is thus, that *„using mathematics we depict highly abstract instances materially and by manipulating this material we can compose statements about the abstract instances"* (Fischer o. J. a, p. 7).

This means that mathematics is not only a formal-operative system, but rather quite essentially a referential system. If we do not take this into account in our mathematics education, we will be transmitting a completely distorted and inadequate image of mathematics.

2. 2 Communication with Experts as an Principle of Orientation for General Mathematics Education

Years ago the German educater and mathematics didactic H. W. Heymann presented a comprehensive concept for general education in teaching mathematics (Heymann 1996). We implemented this concept to control and evaluate our research and development work on the use of CAS in the teaching of mathematics.

Keeping within the framework of this paper it will not be possible for us to deal with Heymann's concept as a whole, rather we plan to concentrate on one aspect in particular, that of the communication between the experts and lay-persons.

The functionality of our highly differentiated, democratic society built up on the division of labor is quite essentially based on an emancipated contact with highly specialized expert knowledge: as mature, responsible citizens we are permanently confronted with statements made by experts which we then must evaluate and judge in order to be able to make our own decisions. As a rule we rely on the professional <u>correctness</u> of these expert statements yet do need to judge their <u>importance</u> for ourselves and for the good of the community. As lay-persons we must be in the position of being able to ask the right questions to the experts, to evaluate their answers and to draw our own conclusions (cf. Fischer 2000, p. 36 - 37).

H. W. Heymann perceives the *„problem of communication between the experts and lay-persons ... to be one of the key problems of a highly differentiated and structured democracy based upon the division of labor“* (Heymann 1996, p. 113). Not only in terms of a professional education schools are playing an important role, the communication between teachers and students is a forming model for the communication between experts and lay-persons (cf. Heymann 1996, p. 114).

R. Fischer picks up on these considerations whereby - in contrast to H. W. Heymann - he primarily concentrates on the professional aspect of communication between experts and lay-persons. He proposes that those persons who have attended institutes of higher learning (high schools and vocational high schools) *„´<u>the more highly educated</u>´“* (Fischer o. J. b, p. 3) in particular should be able to understandably explain the expert opinions and judge their importance (cf. Fischer o. J. b, p. 3f). He suggests that such an „*<u>ability to communicate with experts and with the general public</u>*“ is to be taken as a *„<u>principle of orientation</u>“* for choosing the curriculum at schools of higher education (Fischer o. J. b, p. 3 and p. 4).

R. Fischer identifies the following three areas of competence (Fischer o. J. b, p. 5) as those which are to be acquired:

- *Basic knowledge and basic skills*
  (terms, concepts, forms of representation)
- *Operation*
  (solving problems, proofs, in general: generating new knowledge)
- *Reflection*
  (possibilities, limits and the meaning of terms, concepts and methods)

While the experts in particular have to be competent in the first two areas, R. Fischer considers the areas of competence *basic knowledge* and *reflection* to be particularly important for the generally educated lay-person (cf. Figure 1). Basic knowledge „*is a prerequisite for communicating with experts*“, reflection „*is necessary for judging their expertises*“ (Fischer o. J. b, p. 5).

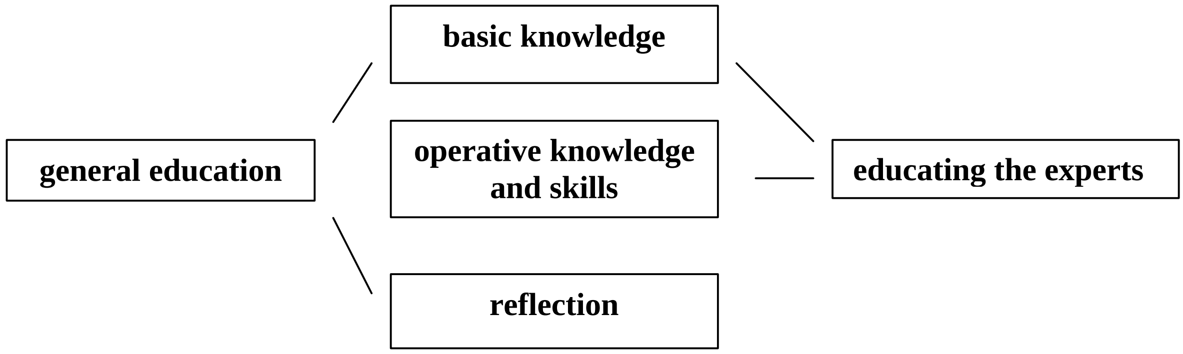

Figure 1: Areas of competence (acc. to R. Fischer o. J. b, p. 5, and 2000, p. 36)

R. Fischer points out that this classification should not be taken as an absolute; neither should the experts be relieved of their responsibility of viewing that which they are doing in a self-critical manner, nor can operation completely be removed from the framework of a general mathematics education. However, the focus and profiles for experts and lay-persons do quite differ. Thus, the consequence for the teaching of mathematics is, in short: „*The reduction of expectations with regard to operations and an increase in the expectations with regard to reflection*“ (Fischer o. J. b, p. 6f).

If one compares the three areas of competence mentioned above with Fischer's „definition“ *mathematics = representing + operating + interpreting* the parallels cannot be denied; basic knowledge of mathematic terms and concepts is transmitted by means of representations (in various forms) and is to be classified as a representative aspect, operative knowledge and skills in the above-mentioned „definition“ are directly contained in operation, interpretation is the local expression of the more global reflection.

Let us summarize:
Not only in view of the main focus of contents but also with regard to the culture of teaching a balanced view of mathematics and the communication between lay-persons and experts can deliver an essential orientation for general mathematics education. The emphasis on the areas of competence of a lay-person with a general education lies in basic knowledge/representation and in reflection/interpretation. (Any further competence in the area of operation are more likely to be tasks required of the experts.)
Schools and teaching are important models for the interaction between experts and lay-persons.

3. The Role of CAS in Mathematics in the Schools

It is most certainly no coincidence that mathematics - as was previously mentioned - is quite often put in the same category as calculation (in a more or less broad sense of the word). Most textbooks are warehouses of exercises in which nothing else is required than a regulated reshaping of mathematic facts given in the form of symbols. And whenever one asks teachers what type of changes they would most urgently like to see in the textbooks, the most frequent answer given is: „More exercises!“ One would suppose that this is then valid not only for the classroom but also for that which is required of the students.
Thus, for most people getting to know mathematics is exactly the same as „doing calculations“ of various sorts and degrees of difficulty.

One can offer comprehensible reasons for this dominance of the operative. So it seems that the operative is the actual purpose of mathematics: only by regularly reworking an exercise does one visibly approach the solution to the problem, are proofs made possible, is new knowledge generated. In the case of work in the schools, it appears that operative knowledge can better fit into a routine, it can be more easily trained and more reliably tested.
In spite of this, the didactic of mathematics is criticizing this dominance of operatives as *„blind calculation actions*“ (H. Winter, as quoted in Fischer/Malle 1985, p. 221) or even as *„conditioning and training something not understood*“ (M. Wagenschein, as quoted in H.-J. Vollrath 1987, p. 376). On behalf of scores of didactics, let me quote H.-J. Vollrath, who has, in our opinion, expressed the essential didactical doubts clearly and concisely:

„The great emphasis on exercises has thrown off the didactical balance. The introductory phases have been reduced, reasoning has been given a subordinate role; understanding has been pushed to the background; routine counts for more than intuition; being able to do something is worth more than knowing how to do it; skills are more important than understanding.“ (Vollrath 1987, p. 376)

Obviously, this dominance of the operative goes against the grain of what we perceive to be the orientation of mathematics education: the balance between representation, operation and interpretation is greatly disturbed by an over-emphasis on the operative; the image we have of mathematics is an unbalanced distortion. Moreover, this concentration on the operative pushes exactly that area of competence to the foreground which – following R. Fischer – we have determined to be more important for training of experts but less important for a general education.

3. 1 CAS and Cultural Coherence

The remarkable resistance of mathematics teaching towards the requirements of mathematical didactics as well as towards pedagogic and didactic concepts of general education can none-theless be explained as a need for cultural coherence.

As teachers we often transmit mathematics in such a manner as we ourselves experienced it (at school or at university; at university mathematics education focuses more strongly on knowledge and comprehension than mathematics at school does but its goal, however, is educating and training experts in mathematics, so the operative becomes much more important than would be necessary within the goals of general education).
Parents expect and demand that their children's mathematics lessons would show only a gentle development in comparison to their own experiences with mathematics, they are not wanting any radical changes. Not only would the parents feel shut out of their children's mathematics education (generation gap!), the parents are afraid that their children will not be learning „real mathematics“ and might suffer the consequences in their future careers or, at the least, that they would encounter problems in their university studies.

New technology, and in particular CAS, are playing a special, ambivalent role in view of cultural coherence of the teaching of mathematics:
CAS are able to function with practically all of the operations which occur in mathematics in school in addition to several which are markedly more than are required at school. These materialized operative knowledge and skills are available for anyone at a current cost of approximately EUR 150.00. Using CAS at school presents a radical change in the traditional teaching of mathematics – and thus suffers from an acceptance problem similar to the didactic demands mentioned before.
On the other hand, not using CAS would lead to a gap in cultural coherence which in this case would be construed as a denial of the technological development of mathematics and of society. (It is hardly justifiable when young people have to spend an exorbitant amount of their time and energy on developing operative knowledge and skills which inexpensive computers or pocket calculators can possess most certainly and more proficiently. Just as it is even more difficult to explain why young people today should obtain the qualities they will need tomorrow with equipment that was old yesterday. - cf. Peschek 1999a, p. 265)

This ambivalence is almost always present and markedly felt in discussions about using CAS in the teaching of mathematics. Some didactics and many teachers try to bridge this gap by only allowing their students the use of CAS after they have proven with pencil and paper that they have obtained operative knowledge and skills.
Others attempt to avoid this gap by using CAS to simulate hand calculations, or even to drill them with the help of computers.
Still others attempt to hide this gap by using examples which hardly could be done without the computer: simulation, three-dimensional representations, different procedures of numerical mathematics, and so on. In the worst-case scenario, even theoretical concepts (such as the Riemann Integral) are being degraded to calculation problems (such as extensive calculations of upper and lower sums).

Our view of the matter differs from the positions mentioned here: we see CAS as a mediator between the didactic requirements for marked reduction of the operative in the teaching of

mathematics on the one hand, and the fears, on the other hand that the potential for problem solving could be lost by the learner. By using CAS operative knowledge and skills are being brought into the mathematics classroom. They are available to students under certain conditions, without having to develop these cognitive knowledge and the skills by the students themselves. In this manner there is a certain amount of breathing space for the development of basic knowledge and basic information, for representation, reflection and interpretation, without limiting the availability of operative knowledge and skills.
Thus we view the ability to work suitably with CAS as a modern form of operative mathematic knowledge and skills: In a CAS-supported mathematics classroom `solve(a*x^2+b*x+c=0,x)` <u>is</u> a solution of the equation $ax^2 + bx + c = 0$ – as well as operating by the rule in a traditional mathematics classroom.

3. 2 CAS and Outsourcing the Operative

We have placed CAS into the role of the mediator. We have attempted, in particular, to keep the differences in operative knowledge and skills with and without CAS to a minimum by depicting the ability to understand the workings of CAS as a modern, materialized form of operative knowledge and skills.
Thus, referring to the cultural coherence in dealing with mathematics in schools, we have taken a more harmonious position on the matter.

Of course, there is a definite and real difference between that knowledge and those skills which we cognitively have available and are directly able to use and those which have been outsourced, being available to us solely by external means (for example, by books, machines, experts). Emancipated application of outsourced knowledge is of great importance not only in our daily lives, but also in the sciences; this is valid in mathematics particularly and specifically because of scientific-theoretical and socio-philosophic reasons (cf. Peschek 1999a, p. 267f, Peschek 1999b, p. 406):

In mathematics we constantly work with the method of <u>outsourcing</u>. This occurs not only in elementary calculations such as division algorithms, but also in the more complex terms and procedures up to and including proofs. One need not know why the division algorithm being applied works in order to get the right answer when dividing; one need not bother with the logical reasoning behind an equivalence transformation, using such an transformation to solve an equation; and one need not recognize the basics of Set Theory of a function in order to successfully set up Calculus. One uses many of these mathematic concepts and procedures as compromised bits of knowledge within mathematics, as <u>modules</u>, of which one needs to know very well the effects and the “interfaces” to outside, to be able to apply them correctly, but not to know their function internally. (Such modules, for which the user don´t know the internal function are frequently called „Black Boxes“.)

In a certain sense outsourcing occurs in mathematics whenever one abstracts relationships from the (outer-mathematic) context and presents them with symbols thus outsourcing the problem in the formal-operative system of mathematics. This outsourcing allows operations to be carried out on a syntactic level, without having any correspondence to the refering context and not being bound to it (therefore, in a certain sense, without „understanding“). These procedures reduce the complexity of the problems actually allowing for economic thinking as well as allowing for solutions and methods of solving which otherwise, without the possibility of transfering into the formal system, would not be so simple to find out.
Such an outsourcing is something genuine for mathematics; it is one of the characteristics of mathematics and it is an essential basis for its performance ability and efficiency. Computers and CAS are for the moment just the last step in the development; they are made possible by materializing mathematically abstract facts in machines as an extension and perfection of outsourcing.

One can immediately establish analogies between scientific-theoretical considerations and socio-philosophic ones: in our high-tech society with its economical division of labor the constant use of „black boxes“ has long become a matter of fact, using "black boxes“ has become an indisputable necessity. Mathematics is taking on a special role:

„*Mathematics is relatively secure, socially accepted, codified knowledge which, in a measured amount, allows for a separation between understanding and doing* ..... (it) *owes its high social relevance to the fact that, in utilizing outsourcing, it even works when the user has no idea anymore as to why*“. (Peschek 1999b, p. 406)
(More detailed and encompassing discussions on this matter can be found, for example, in Fischer 1991, Maaß/Schlöglmann 1988 or Peschek 1999a).

An elaborate image of mathematics, of its characteristics, its thought and work processes and its socio-cultural importance would include outsourcing as an important scientific-theoretical and socio-philosophical aspect; CAS and computers in general as modern and recognizable examples could simplify reflection and thematic this basic property of mathematics. In this sense we have formulated the principle of outsourcing some years ago:

„*Mathematics education should use modern means for introducing and applying mathematic concepts; in particular, it should also attempt, to* ***outsource*** *operative knowledge and operative skills in these modern means as far as this is possible in a didactically sensible way. In doing so, it must be considered that those learning enables to*

- *efficiently use of mathematical black boxes*
- *judge the prerequisites, effects, range and limits of the black boxes used and*
- *inspect the scientific theoretical and the social importance of using mathematical black boxes.*

I call this didactical principle the ***principle of outsourcing***.“ (Peschek, 1999b, p. 407)

The principle formulated as such clearly shows that outsourcing is by no means something where one can do without understanding mathematics. The opposite is true: operative knowledge and operative skills should only be outsourced to CAS in as far as this outsourcing seems to make sense didactically (which, in any case, needs to be considered by, explained to and discussed even with the students). In any case there are certain prerequisites, effects, ranges and limits of operative modules to be reflected upon in order to guarantee an understanding and efficient use of the modules. And last but not least, outsourcing should also be experienced and understood as a fundamental characteristic of mathematics and as a constitutive aspect of its social importance.
The computer (CAS) can serve as a very clear example and model for such experiences.

3. 3 The Use of CAS is Communication with Experts

According to R. Fischer, we placed the area of competence for operative knowledge and skills primarily to the experts. This turns out to be exactly the same area of competence which could be outsourced most completely to CAS. In this sense we could view CAS as an electronic model of a mathematic expert.
We will admit that CAS is not going to be a particularly good mathematic expert, being too limited and percise in its communication with us users. Its range of basic knowledge in mathematics is much too narrow, its ability to present and interpret and even its work in the operative will be bound to disappoint us now and again. CAS cannot be a substitute for human experts (and particularly not for teachers). But by using CAS in the communication between people and machines there are elements which are also important for the communication between lay-persons and human experts: A targeted and profitable interaction with CAS presumes thorough basic knowledge in mathematics; it demands very exact ideas of the fundamental possibilities and limits, as well as estimates of the local abilities of CAS (sometimes these could also be „experimentally“ determined in the interaction with CAS); it requires the willingness and the ability to ask the right questions, to be precise when formulating one's own questions and contemplations and to present these symbolically in a form which can be interpreted by CAS. And finally it requires a check, a comprehensible interpretation and evaluation of the answers CAS has provided.
Whenever CAS users (students) are working within corresponding social forms, something else is happening: the transmission of the answers provided by CAS to other lay-persons, the discussion of these answers among the lay-persons and the negotiation process of their evaluation as well as of any further questions for the experts might arise. All in all, these are essential elements of what R. Fischer (o. J. b, p. 4) designated to be communication with the general public.

Due to the reasons briefly sketched out here, we see the use of CAS as a model for the communication between mathematics experts and lay-persons that is also didactically and pedagogically interesting and useful.

4. Basic Knowledge and Basic Skills

What can neither be objectively nor obviously derived from the conceptions of mathematics and the teaching of mathematics which have been sketched here (or also others which have been presented) is which basic knowledge and which concrete basic skills should be taught at school. This has to be set by the society and experts in a negotiating process.
However, the concepts presented here can represent a framework for such a process of negotiations in which the suggestions and arguments presented are able to be checked, categorized, classified, evaluated, weighed and set up for comparison.
In addition to this, they can also be a kind of orientation for the areas in which basic knowledge and basic skills can primarily be found. Without claiming completeness, without weighing and establishing any further argumentative network to the previously sketched concepts, we would like to mention several areas which, from our point of view, could be seen as important examples of basic knowledge and representation as well as of interpretation and reflection:

- Knowledge of fundamental mathematic concepts, understanding their local meanings and interpretations and knowing of several cases in which these concepts are typically used.
- Knowledge of various forms of representing abstract mathematic facts (verbally, in tables, graphically, symbolically), recognizing their advantages and disadvantages and „ qualifications for translating“.
- Ability to use materialized operative knowledge and skills in a comprehensible and controlled manner (whereby we ourselves prefer those systems which allow CAS to be permanently available to the students).
- Insight into the fundamental ideas of mathematic issues and their parallels in our everyday thinking processes.

We would like to articulate this in the form of a few concrete (exam) exercises on different levels, in which the relevant knowledge and skills were taken into consideration:

Example 1: Increases in wages can take the form of a fixed amount f or in percent p of the gross wages (or they can also be some complicated mixture of the two).
Let b be the function, which assignes to each gross wage B_0 the amount after the increase.

a) Give the function equation for both forms of wage increase!
What kind of functions are being dealt with here?
b) Sketch the graphs of both functions!
c) Explain using function graphs, who will have what advantage by receiving which type of wage increase!

d) At which amount income would you prefer a wage increase of 2% to a wage increase of EUR 50.00?

Example 2: The common formula for calculating interest is $K_t = K_0 \cdot r^t$.

a) Show that interest charged separately on two capital resources leads to the same result as charging interest for both capital resources together!

b) Show that interest charged separately on two periods in succession leads to the same result as charging interest for both time periods as a whole entity.

c) Interest charged on credits is chiefly based on the assumption that one can move people in an organized monetary society based on the division of labor to work together in such a way that (due to synergy effects) the entire work performance possible is larger than the sum of the individualized work. The same is valid for the duration of the interest charged.
How does this basic assumption perform under the proven properties of the interest formulas shown in a) and b)?
In what way can (and in practice is) this problem „be solved“?

Example 3: Let K(t) be the amount of the working population of a given country, who were off due to illness exactly t days in the year 1999.

a) What does $\int_0^{365} K(t) \cdot dt - \int_1^{365} K(t) \cdot dt$ mean?

b) Submit the average duration of sick leave of the employed population of this country for the year 1999 by using integrals! Which central measurement is being used here?

Example 4: Let $p_n: x \rightarrow p_n(x)$ be the price-market-function of a product, let p_0 be its sales price, let x_0 be the selled amount (for price p_0).
The content of the area marked in Figure 2 is called „consumer's surplus“.

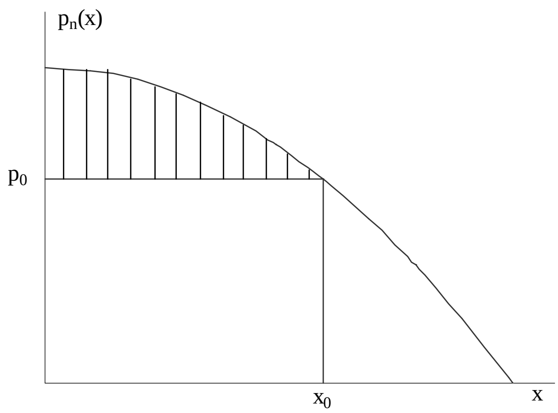

Figure 2: Consumer’s surplus

a) Consider what the consumer's surplus depicts?
b) Whenever a new product is introduced, one frequently observes gradual price reductions. What does this mean with regard to consumer's surplus?
c) Calculate the consumer's surplus for $p_n(x) = 50-\sqrt{x}$ and $x_0 = 900$!

Example 5: At the run-off elections on August 27, 1992 Ion Illiescu (of the FSN - Frontul Salvirii Nationale) was elected to be the Romanian Head of State with approx. 62% of the valid ballots.

One year later, „Romania Libera“, the third-largest daily newspaper in the country asserted that Illiescu would achieve the same result were the elections to be held at that current time.

This was vehemently denied by the FSN as well as by almost all of the opposition parties. One of these opposition parties interviewed 100 eligible Romanian voters and declared gloatingly, that only 52 of the 100 persons questioned would have voted for Illiescu this time around. This would mean that one could not be certain that Illiescu even had a majority mandate in the country.

What do you think about this survey and the statements made by the newspaper „Romania Libera“ and the opposition?

We think it is clear that when using such examples it is of little significance to us whether the students are doing the operative work with or without CAS.

5. Closing Remarks

There is really no mystery with and around CAS:
CAS could operate with symbolically presented mathematic objects in such a manner as does correspond to operating by rules within the field of mathematics. This allows for outsourcing the operating by rules to a CAS. That is the reason that CAS was developed and in this sense CAS have become a modern materialization of (constitutive parts of) mathematics and simultaneously they are the last step in their trivialization.
A mathematics classroom which keeps to the dominance of the operative – regardless of the use of CAS or not – seems to us to be not only quite conservative, it runs the risk of becoming trivial itself.
Many papers and projects on the use of CAS have shown us that concentrating on operatives – and so the risk of trivializing the teaching of mathematics – has been on the increase rather than having decreased at all. Let´s stop this development, let´s stop to compete with CAS but use it sensible and let´s try to find to a modern general mathematic education for our students.

References

Fischer, R. (1990): *Längerfristige Perspektiven des Mathematikunterrichts*. In: mathematica didactica 13 (1990), S. 38 - 61.

Fischer, R. (1991): *Mathematik und gesellschaftlicher Wandel*. In: Journal für Mathematik-didaktik 91/4, S. 323 - 345.

Fischer, R. (1999): *Technologie, Mathematik und Bewußsein der Gesellschaft.* In: Kadunz, G. u. a.: Mathematische Bildung und neue Technologien. B. G. Teubner, Stuttgart-Leipzig, S. 85 - 102.

Fischer, R. (o. J. a): *Mathematik anthropologisch: Materialisierung und Systemhaftigkeit*. Unv. Manuskript, 13 S.

Fischer, R. (o. J. b): *Höhere Allgemeinbildung*. Unv. Manuskript, 15 S.

Fischer, R. (2000a): *Universitäre Allgemeinbildung*. In: Arnold, M. und Fischer, R.: Studium Integrale. Springer,Wien, S. 35 - 40.

Fischer, R. und Malle, G. (1985): *Mensch und Mathematik. Eine Einführung in didaktisches Denken und Handeln*. B. I. Wissenschaftsverlag, Mannheim-Wien-Zürich.

Heymann, H. W. (1996): *Allgemeinbildung und Mathematik*. Studien zur Schulpädagogik und Didaktik, Band 13. Beltz, Weinheim-Basel.

Maaß, J. und Schlöglmann, W. (1988): *Die mathematisierte Welt im schwarzen Kasten – die Bedeutung der Black Boxes als Transfermedium*. In: Bammé, A. u. a. (Hrsg.): Zivilisation und die Transformation des Wissens. Profil, München, S. 379 - 398.

Peschek, W. (1999a): *Mathematische Bildung meint auch Verzicht auf Wissen.* In: Kadunz, G. u. a.: Mathematische Bildung und neue Technologien. B. G. Teubner, Stuttgart-Leipzig, S. 263 - 270.

Peschek, W. (1999b): *Auslagerung als didaktisches Prinzip eines computerunterstützten Mathematikunterrichts.* In: Neubrand, M. (Hrsg.): *Beiträge zum Mathematikunterricht 1999.* Franzbecker, Hildesheim-Berlin, S. 405 – 408.

Schneider, E. (1999): *Mathematische Bildung trotz des und mit dem TI-92*. In: Kadunz, G. u. a.: Mathematische Bildung und neue Technologien. B. G. Teubner, Stuttgart-Leipzig, S. 287 - 294.

Schneider, E. (2000): *Teacher Experiences with the use of a CAS in a Mathematics Classroom*. In: The International Journal of Computer Algebra in Mathematics Education 7/3.

Vollrath, H.-J. (1987): *Störungen des „didaktischen Gleichgewichts“ im Mathematik-unterricht*. In: Der mathematisch-naturwissenschaftliche Unterricht 40/6, S. 373 - 378.