HOW TO IDENTIFY BASIC KNOWLEDGE AND BASIC SKILLS IN CAS-SUPPORTED MATHEMATICS EDUCATION?

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Abstract
In order to have a rational discussion about a subject it is not sufficient just to have an opinion – one needs arguments based on a (theoretical) frame of reference, allowing for classification and for a rational assessment of the arguments.
This paper discusses descriptive and normative perspectives of mathematics and of (general) mathematics education which can be found in the didactic literature. As we see it, these in turn, are well suited to contemplate and to evaluate the use of Computer Algebra Systems (CAS) as well as suggestions for basic knowledge and basic skills in a modern technology-supported mathematics classroom.

1. Mathematics as an Interplay between Representing, Operating and Interpreting

Current conception of mathematics is, for the most part, closely coupled to those of calculations. Calculation means the reshaping of mathematic facts according to rigidly formulated rules. R. Fischer and G. Malle designate this „generalized calculation“ as "operating by rules“ (Fischer/Malle 1985, p. 221).

Operating by rules presumes that the *representation* of the fact occurs in such a way that operating by rules is made possible at all and it only makes sense where the *interpretation* of the results of the reshaping provide relevant information in the context of the original fact which was not available before having carried out the operation.

One can view this interplay of representing, operating by rules and interpreting in many exercises outside of or within mathematics: a fact is *represented* by an arithmetic expression, as an equation or a system of equations and inequations, as an integral or as a differential equation. *Operating by rules* (carrying out arithmetic operations, solving equations, systems of equation or inequation, determining a derivative, calculating an integral and so on) leads to a new representation of the fact. The *interpretation* of this new representation delivers new information about the given context (for example: the amount of a bill, an optimal production program, local extremes, the content of the area underneath a curve).

These considerations lead R. Fischer to the following „definition of mathematics“ (Fischer 1990, p. 38), as simple as it is encompassing:

Mathematics = representing + operating + interpreting

In mathematics we have *abstract* (not directly perceptible to the senses) *relationships*, which are represented (materialized) in a way which allows for regular reshaping: "Mathematics is the material, symbolic representation of abstract facts which cannot be directly perceived by the senses offering the possibility of regularized reshaping" (Fischer 1999, p. 90).

This „definition“ refers to an obligation of mathematics to recognize (or invent) a representation for an abstract fact which allows for operating by rules; interpretation then means the explanation by means of facts represented in symbols (or also in another way). „Doing mathematics consists of much more than the greater part of an interaction between the person and the form of representation (whether on the paper or on the screen). One alters the form of representation, contemplates it, alters it again, contemplates it, and so on and so on" (Fischer o. J. a, p. 7).

If we do not take this into account in our mathematics education, we will be transmitting a completely distorted and inadequate image of mathematics.

2. Communication with Experts as a Principle of Orientation for General Mathematics Education

The functionality of our highly differentiated, democratic society built up on the division of labor is quite essentially based on an emancipated contact with highly specialized expert knowledge: as mature, responsible citizens we are permanently confronted with statements made by experts which we then must evaluate and judge in order to be able to make our own decisions. As a rule we rely on the professional *correctness* of these expert statements yet do need to judge their *importance* for ourselves and for the good of the community. As lay-persons we must be in the position of being able to ask the right questions to the experts, to evaluate their answers and to draw our own conclusions.

Not only in terms of a professional education schools are playing an important role, the communication between teachers and students is a forming model for the communication between experts and lay-persons (cf. Heymann 1996, p. 114).

R. Fischer picks up on these considerations whereby he primarily concentrates on the professional aspect of communication between experts and lay-persons. He proposes that those persons who have attended institutes of higher learning (high schools and vocational high schools) in particular should be able to understandably explain the expert opinions and judge their importance (cf. Fischer o. J. b, p. 3f). He suggests that such an „ability to communicate with experts and with the general public“ is to be taken as a „principle of orientation“ for choosing the curriculum at schools of higher education (Fischer o. J. b, p. 3 and p. 4).

R. Fischer identifies the following three areas of competence (Fischer o. J. b, p. 5) as those which are to be acquired:

- **Basic knowledge and skills**

(terms, concepts, forms of representation)

- **Operation**

  (solving problems, proofs, in general: generating new knowledge)

- **Reflection**

  (possibilities, limits and the meaning of terms, concepts and methods)

While the experts in particular have to be competent in the first two areas, R. Fischer considers the areas of competence *basic knowledge* and *reflection* to be particularly important for the generally educated lay-person. Basic knowledge „is a prerequisite for communicating with experts", reflection „is necessary for judging their expertises" (Fischer o. J. b, p. 5). Thus, the consequence for the teaching of mathematics is, in short: „The reduction of expectations with regard to operations and an increase in the expectations with regard to reflection" (Fischer o. J. b, p. 6f).

3. The Role of CAS in Mathematics in the Schools

As previously mentioned, for most people getting to know mathematics is exactly the same as „doing calculations" of various sorts and degrees of difficulty. One can offer comprehensible reasons for this dominance of the operative, but obviously, this dominance of the operative goes against the grain of what we perceive to be the orientation of mathematics education: the balance between representation, operation and interpretation is greatly disturbed by an over-emphasis on the operative; the image we have of mathematics is an unbalanced distortion. Moreover, this concentration on the operative pushes exactly that area of competence to the foreground which – following R. Fischer – we have determined to be more important for training of experts but less important for a general education.

We see *CAS as a mediator* between the didactic requirements for marked reduction of the operative in the teaching of mathematics on the one hand, and the fears, on the other hand that the potential for problem solving could be lost by the learner. By using CAS operative knowledge and skills are being brought into the mathematics classroom. They are available to students under certain conditions, without having to develop these cognitive knowledge and the skills by the students themselves. In this manner there is a certain amount of breathing space for the development of basic knowledge and basic information, for representation, reflection and interpretation, without limiting the availability of operative knowledge and skills.

3. 1 CAS and Outsourcing the Operative

Of course, there is a definite and real difference between that knowledge and those skills which we cognitively have available and are directly able to use and those which have been *outsourced*, being available to us solely by external means (for example, by books, machines, experts). Emancipated application of outsourced knowledge is of great importance not only in our daily lives, but also in the sciences. Mathematics is taking on a special role: „Mathematics is relatively secure, socially accepted, codified knowledge which, in a measured amount, allows for a separation between understanding and doing ... (it) owes its high social relevance to the fact that, in utilizing outsourcing, it even works when the user has no idea anymore as to why“ (Peschek 1999b, p. 406; more detailed and encompassing discussions on this matter can be found, for example, in Fischer 1990, Maaß/Schlöglmann 1988 or Peschek 1999a). So, outsourcing is something genuine for mathematics; it is one of the characteristics of mathematics and it is an essential basis for its performance ability and efficiency. Computers and CAS are for the moment just the last step in the development; they are made possible by materializing mathematically abstract facts in machines as an extension and perfection of outsourcing.

In this sense we have formulated *outsourcing* as an important *didactical principle* in a modern mathematics teaching (cf. Peschek 1999b, p. 407).

3. 2 The Use of CAS is Communication with Experts

According to R. Fischer, we placed the area of competence for operative knowledge and skills primarily to the experts. This turns out to be exactly the same area of competence which could be outsourced most completely to CAS. In this sense we could view CAS as an electronic model of a mathematic expert.

We will admit that CAS is not going to be a particularly good mathematic expert, being too limited and percise in its communication with us users. Its range of basic knowledge in mathematics is much too narrow, its ability to present and interpret and even its work in the operative will be bound to disappoint us now and again. CAS cannot be a substitute for human experts (and particularly not for teachers). But by using CAS in the communication between people and machines there are elements which are also important for the communication between lay-persons and human experts: A targeted and profitable interaction with CAS presumes thorough basic knowledge in mathematics; it demands very exact ideas of the fundamental possibilities and limits, as well as estimates of the local abilities of CAS; it requires the willingness and the ability to ask the right questions, to be precise when formulating one's own questions and contemplations and to present these symbolically in a form which can be interpreted by CAS. And finally it requires a check, a comprehensible interpretation and evaluation of the answers CAS has provided.

Whenever CAS users (students) are working within corresponding social forms, something else is happening: the transmission of the answers provided by CAS to other lay-persons, the discussion of these answers among the lay-persons and the negotiation process of their evaluation as well as of any further questions for the experts might arise.

All in all, these are essential elements of what R. Fischer (o. J. b, p. 4) designated to be communication with the general public.

Due to the reasons briefly sketched out here, we see the use of CAS as a model for the communication between mathematics experts and lay-persons that is also didactically and pedagogically interesting and useful.

4. Basic Knowledge and Basic Skills

What can neither be objectively nor obviously derived from the conceptions of mathematics and the teaching of mathematics which have been sketched here (or also others which have been presented) is which basic knowledge and which concrete basic skills should be taught at school. This has to be set by the society and experts in a negotiating process.

However, the concepts presented here can represent a framework for such a process of negotiations in which the suggestions and arguments presented are able to be checked, categorized, classified, evaluated, weighed and set up for comparison.

In addition to this, they can also be an orientation for the areas in which basic knowledge and basic skills can primarily be found. Without claiming completeness, without weighing and establishing any further argumentative network to the previously sketched concepts, we would like to mention several areas which could be seen as important examples of basic knowledge and representation as well as of interpretation and reflection:

- Knowledge of fundamental mathematic concepts, understanding their local meanings and interpretations and knowing of several cases in which these concepts are typically used.

- **Knowledge of various forms of representing abstract mathematic facts (verbally, in tables, graphically, symbolically), recognizing their advantages and disadvantages and „qualifications for translating“.**
- Ability to use materialized operative knowledge and skills in a comprehensible and controlled manner (whereby we ourselves prefer those systems which allow CAS to be permanently available to the students).
- Insight into the fundamental ideas of mathematic issues and their parallels in our everyday thinking processes.

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