

# FROM THE EUCLIDEAN TOOL TO THE COMPUTER ALGEBRA SYSTEM

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## ***Abstract.***

*The appearance of C.A.S. which can be executed from Pc's provides us with a lot of new possibilities in our teaching work. We show how we interpret C.A.S. can be used in order to save the students making heavy and repetitive calculus and therefore we can focus their work in the knowledge we understand important, as well as, it lets the teacher could include in the exams questions close to the reality.*

## **Introduction:**

In any discipline, teaching and learning methods are conditioned by the available tools which is also the case for Mathematics. In this way, two thousand years ago, Geometrical aspects of Mathematics were studied by means of Euclidean Geometry (Ruler and Compass) and calculus were made by using the Calculus Ruler which has been used until the sixties. In particular, it has been necessary to make complex calculus in order to carry out any mathematical study. This feature has distracted the student from the mathematical theoretical basis of his study, and so, from his main goal.

Since the C.A.S. frees the student to deal with heavy calculus, it is possible to increase the number of questions per exam, as well as, to propose more complete and closer to the reality questions. On the other hand, this technology allows the student to operate with algebraic expressions by applying directly the theoretical results and obtaining interesting relations

between the inputs and the outputs of some problems, provided he works with parameters as inputs.

In this sense, we have developed some *Mathematica* packages and notebooks over these subjects:

- Function Graphic Representations.
- Error Calculus.
- Torsorial Statics.
- Network Analysis.
- Dynamical System Analysis.

Also, we have described both the basic knowledge for each of them (without which the software becomes useless), and the knowledge to gain.

Our pupils:

They are studying early courses of Computer Science or of Industrial Engineering studies. These packages are oriented on such level of mathematical background, with two exceptions:

- Function Graphic Representations (Secondary School)
- Dynamical System Analysis (final university courses)

The packages over Network Analysis and Torsorial Statics mainly deal with Physics Science but they also work the mathematical concepts.

Package Function Graphic Representations:

This package provides an environment in which it is possible to study the graph of a function, by using functions like *firstderivativeroots* that eliminate a lot of calculus which were tested in previous courses. Also, the package includes the function *paintbetween[a,b]* that allows the user to correct the results (to make self-evaluation).

Theoretical background: Limit Calculus, Solving Equations, Increasing and Decreasing Intervals, Differential Calculus.

Knowledge to gain: Definition Dominions, Asymptotic Lines, Graph of a Function.

Package Error Calculus:

This package provides an environment in which it is possible to learn how to write any value in the correct format (taking into account its error). This package, mainly but not only, eliminates the calculus corresponding to partial derivatives.

Theoretical background: Partial Derivatives (Calculus), Direct and Indirect Measures, Absolute and Relative Errors, Accepted Value (Physics).

Knowledge to gain: Error Propagation Theory, Writing in an adequate form any value with its absolute error.

Package Network Analysis:

This package provides an environment in which it is possible to analyze electrical networks in continuous current by applying the Kirchoff's Laws. Also, it can be made an interesting study which relates the problem data with the solutions in a formal way.

Theoretical background: Linear Systems Theory, Cramer and Gauss Methods (Algebra), Kirchoff's Laws, Loops and Nodes Methods (Physics).

Knowledge to gain: Understand the practical meaning of the Kirchoff's Laws, Applying in a properly way these methods in order to solve networks in continuous current.

Package Torsorial Statics:

This package provides an environment in which it is possible to analyze any static system of forces from the torsorial (by using torsors, also called twistors) point of view.

Theoretical background: Linear Systems Theory (Algebra), Sliding Vectors (Geometry), Static of the Rigid Solid (Physics).

Knowledge to gain: How to apply the free-body diagram, how to solve problems on Static by using Torsorial Algebra.

Package Dynamical System Analysis:

This program finds and identifies points of bifurcation of families of maps on  $\mathbf{R}$  or on  $\mathbf{R} \times \mathbf{R}$ .

Theoretical background: Basic Topology, Differential Calculus, Implicit and Inverse Function Theorem, Relative Extremes of functions, Topics on Dynamical Systems.

Knowledge to gain: Topological Equivalences on families of maps, Bifurcations on families of maps on  $\mathbf{R}$  or on  $\mathbf{R} \times \mathbf{R}$ .

Conclusions and Future Work:

We have presented some *Mathematica* software which can be used by the students, both in order to solve problems that test specific contents (without repetitive calculus) within a reasonable amount of time and in order to make self - evaluation (after a little remaking of the presented software).

Our future work is focused on implementing notebooks (which are at your disposal in the Public Site of our Shared Computer Work Space) taking into account the Computer Oriented Education principles.

**See full paper in: <http://csimbolico.rediris.es/pub/spanish.cgi/0/125311>**