AN INTRODUCTION TO DIFFERENTIAL CALCULUS USING *MATHEMATICA*

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Abstract.
This paper describes how differential calculus was introduced to a group of International Baccalaureate students taking the Maths Studies option at Vienna International School. This introduction covers the concept of the gradient of a curve at a point, an informal concept of a limit, maxima and minima, differentiation of a polynomial and the application to simple optimisation problems. The paper o utlines the nature of the students, the specific learning objectives and briefly the outcomes. The rational for using a technology assisted strategy is also discussed. The nature and advantages of Mathematica are mentioned.

The International Baccalaureate is an external examination intended to be valid for university entrance in as many countries as possible. The syllabus, as well as the examinations, are set externally, that is, without direct input from the teacher. Students taking the IB must take at least six subjects, one of which must be mathematics. There are various mathematics options, the lightest of which is Maths Studies. It is therefore true to say that in general students taking Maths Studies are most interested and motivated in subjects other than mathematics. The students in this case were the most mathematically able of those taking Maths Studies.

The Maths studies syllabus has a calculus option. The learning objectives listed below are derived directly from the Maths Studies syllabus. It should be mentioned that the students wishing to study in Austria must obtain “Matura Equivalence”, which means, amongst other things, that they must have studied calculus to some extent. The learning objectives that we worked from follow:

The students should be able to:

1) Explain that the gradient of a function $f(x)$ at $x=a$ is approximated by the gradient of of the line through $f(a)$ and $f(a+d)$, and state that the approximation is better when d is smaller.
2) State that the gradient of a quadratic function is a linear function.
3) Identify from the graph of a function when the gradient is -ve, 0 or +ve.
4) Differentiate polynomial functions.
5) Formulate simple mathematical models of optimisation problems and use differential calculus to solve them analytically.

Of the above we identified objective 1) as being most difficult to demonstrate using conventional chalkboard techniques. Put briefly, we wish to show several straight line approximations to a curve and demonstrate which is the better approximation, this leads to a rather untidy and unclear diagram on the chalkboard. The desire for particularly clear diagrams here made *Mathematica* rather than graphics calculators the chosen tool. Below we describe how *Mathematica* was used to address this objective and others.

Mathematica is a symbol manipulator with many features, three of which are particularly relevant to the subject of this paper.

1) *Mathematica* has particularly good graphics capabilities. Naturally students may graph functions.
2) It has a notebook facility. Students and teachers are able to embed textual explanations in a document otherwise composed of *Mathematica* commands, results and graphics.

3) It has a programming language. We do not ask the students to use this but it allows the teacher to produce specific functions to be used by the students.

We have twenty copies of *Mathematica* 2.2. One of these is installed on a teacher machine with an overhead facility in a computer laboratory, the rest are installed on the student machines in this lab. For this particular group, we found that the timetable allowed access to the computer lab for one 1 hour 20 minute period per week. Below we describe particularly how we used this resource.

We have used the programming facility to produce a function, called GradPlot that does the following: The user supplies a function f, a point on the domain a and a "run" d. GradPlot then produces a graph of f, marks the points $(a,f(a))$ and $(a+d,f(a+d))$ with small coloured squares, draws a straight line between these points and displays the numerical value of the gradient of this line. This function is embedded in a notebook explaining it's use and suggesting student activities using this function. We start the computer lab activity by demonstrating this function with, say, $f=x2$ and $a=1$. We then display the graphic with varying values of d and discuss with the students which value of d gives the best approximation to a tangent at a point and so the best approximation to the gradient of the function at the point. The students readily accept that the smaller d, the better the approximation. The students then use this function to explore how the gradient of $x2$ depends on x. For this activity they typically use a value of $d = 0.000001$ and record and plot the gradient value approximations manually. They "discover" that the gradient is very close to $2x$. During this activity we ask the students to comment on the sign of the derivative and the nature of the curve at that point. They can use this technique to explore the derivatives of other powers of x and try to discover the general rule.

The students used *Mathematica* for other activities that are described briefly below. During all these activities the students are required to record results, conclusions and explanations either with pen and paper or electronically.

We also have a self produced function Diff2Plot that plots a function and it's derivative on the same graph. The students used this function to investigate the derivatives of higher powers of x, of sums of powers of x and of powers of x multiplied by a constant. Again we asked the students to relate positive, negative and zero values of the derivative to the nature of the original curve. We looked at the derivatives of general quadratics and emphasised the ideas of turning points.

The students were introduced to the *Mathematica* D function that gives the expression for a derivative when the function is supplied. They were also introduced to the Solve function that solves an equation. They used these functions to attempt optimisation problems. In their answers to these problems they were asked to supply a full explanation of their strategy and reasoning.

It should be added that all these computer activities were augmented by classroom lessons where ideas were discussed, reinforced, formalised and applied.

At the end of this activity we gave the students a test. This was a pen and paper test without access to the computer, and intended to reflect the style of questions that the students may expect in the final IB Maths Studies examination. The results of this assessment have not yet been fully analysed, but a first reading is encouraging although pointing out areas of our strategy where improvement is necessary. We intend to repeat this activity with another group of students, and possibly develop the use of *Mathematica* to help teach other areas of mathematics.