USING DATA TO PROMOTE MATHEMATICAL UNDERSTANDING

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Abstract:
Hand-held data collection technology allows for access to real-world data collection – at any other time and almost any place. Is the use of data, and its collection, desirable to the mathematical leaning process? The answer is a resounding yes! Not only can we teach significant mathematical ideas in the process; we also help our colleagues in the sciences. When done correctly, students actively involved in the data collection process take ownership of the data and of the mathematical learning that follows. Ownership, in turn, leads to understanding. Teachers must also be able to ask good questions to help guide and direct students as they start an activity. If used as a demonstration in class, the teacher must be able to lead students to the desired mathematical outcome - understanding a mathematical concept.

Today, we seem to find our students showing less interest in the mathematics we teach. While it may seem this way, the masses of students have, for a long time, been somewhat disinterested. This may be because we have not been able to instill our passion for mathematics in the way we teach. Our students may not see the beauty and excitement in the derivation of the quadratic formula. They may not appreciate the marvelous and simple development of the properties of logarithms from the laws of exponents as we do.

One possible solution is to teach as many topics as is reasonable by introducing them in the context of a real-world situation. Many students ask us where they will use the mathematics we teach. Teaching in context answers this question right up front, and, at the same time, provides motivation for the mathematics to be taught. Further, and more importantly, contextual situations can bring a mathematical understanding of the mathematical concepts heretofore unattainable by traditional pedagogy.

The Calculator Based Laboratory 2 ™ (CBL 2) is a teaching tool that gives us ample situations that can be used to assist in teaching for understanding. Following are a very limited set of examples of activities that promote mathematical understanding through the use of data and data collection. You will also note references below to developing a model for the data collected using the connections between function parameters and the related function behaviors. Using this connection is a powerful teaching tool as well. Teaching experiences of the author using the ideas presented have been exceptional.

Finally, the use of leading questions is a method for piquing the interest of students. Further, it is used to direct the attention of the students to the topic at hand. You can take the students from what they know and understand about the topic and lead them to a proper and more complete understanding.

Mathematical Topic: Asymptotic behavior- vertical asymptote

Equipment needed:	CBL 2™, TI calculator, DataMate software app, Vernier pressure sensor.
Information:	The CBL 2™ will collect volume (in cc) and pressure (in ATM or other units) data for the air secured in a 20-cc syringe.
Mathematics	
Pre-requisites:	Students should have a little experience in developing linear and quadratic models from data sets using their understanding of the connections between function parameters and the resulting function behaviors. At the introductory level, there is no need for students to use regression models. This activity is intended to introduce students to their first function that displays vertical asymptotic behavior.

Leading Questions:

– As you cause the volume to approach zero with your hand, what do your senses tell you about the pressure?

- Do you think you can change the volume to 0 cc in the syringe? (without damaging the equipment)
- If you could change the volume to 0 cc, what would happen to the pressure?
- Do you think the relationship between volume and pressure is linear? That is, for any ΔV and related ΔP is the ratio $\frac{\Delta P}{\Delta V}$ constant?
- Might the relationship between volume and pressure be quadratic? Explain.
- Describe a trend (pattern) you see in the **change in the pressure** on the sensor as you **change** the volume from 20 to 19 cc, from 19 to 18 cc, from 15 to 14 cc, from 10 to 9 cc, from 6 to 5 cc. (The pressure is continuously displayed by the DataMate software.)
- As the volume approaches zero, describe the general behavior of the rate of change of the pressure.
- Finally, it is the time to collect and store the data points. Perhaps at 20, 18, 15, 13, 10, 8, and 5 cc's. Be sure to let the pressure sensor stabilize at each volume before collecting the data.
- What might the pressure be at 3 cc of volume? 2 cc of volume? (This assumes that most people cannot get the volume to 2 or 3 cc. It may also provide motivation for the need of the symbolic form of the relationship.)
- Graph the data. Does the graph behave as you expected?
- Summarize the behavior of the graph of the relationship as the volume approaches zero.
- Do you notice any pattern developing between the *x* and *y*-coordinates? (volume-pressure)

- Multiply the x and y-coordinates together and store the results in L_3.
- Do you notice any pattern developing between the x and y-coordinates now?
- Make a suggestion for a symbolic model of the data relationship and graph it.
- How does the graph of the model relate to the data points?

You are now ready to analyze the behavior, and functions with this behavior, using a more formal mathematical approach.

Mathematical Topic: Piece-wise defined functions

Equipment needed:	CBL 2™, temperature probe, TI calculator, a hot hand
Information:	The CBL 2 is going to collect time-temperature (either °C or °F) data as the temperature probe is heated in my hand for 30 seconds and then cooled in the air for 30 seconds.
Mathematics	
Pre-requisites:	Students should have some experience in developing models from data sets using their understanding of the connections between function parameters and the resulting function behaviors. There is no need for students to use regression models. This activity is intended to introduce students to the need for a function that does not behave as do simple elementary functions typically studied in pre-calculus courses.

Leading Questions:

- Do you think the heating portion of the time-temperature graph will display similar behaviors as the cooling portion?
- Will the graph be symmetric to a vertical line through 30 seconds? Why? *Collect data now.*
- After seeing the visualization of the data set, what function have you studied that behaves in this fashion?
- Do you think it is possible to have a single function that could be used to model this data set?
- What kind of function is suggested by the data for the heating portion of the data set? Find it.

- What kind of function is suggested by the data for the cooling portion of the data set? Find it.
- If you add the two functions above, will the graph of the sum be a good model of the data?
- Is it important to control the domain of either model? How can you control the domain of a function?
- What is the domain of the function $y = 2x - 3 + 0\sqrt{x-1}$?
- Is the graph of $y = 2x - 3 + 0\sqrt{x-1}$ linear?
- Propose a solution to the problem of not being able to model the data with a single elementary function.

You are now ready to analyze piece-wise defined functions, using a more formal mathematical approach.

Mathematical Topic: Maximum, increasing, decreasing, zeros

Equipment needed:	CBR, (preferably used with the CBL 2) TI calculator, light-weight ball (soft)
Information:	The CBR will collect time-distance data, as a ball is tossed straight up above the CBR lying directly below on the floor. Time will be in seconds and distance in meters.
Mathematics	
Pre-requisites:	Students should have some experience in developing linear and quadratic models from a data set using their understanding of the connections between function parameters and the resulting function behaviors. There is no need for students to use regression models. This activity is intended to promote understanding of the topics. However, this activity (excluding the modeling part) might be used with no pre-requisites other than a basic recognition that some data sets when graphed display a variety of common behaviors. In this case, it can be used as the first contact students have with the concepts of increasing, decreasing, maximum, and zeros. Actually, this activity contains too many objectives for a good lesson plan and is used here as a demo only.

Leading Questions:

- As I toss the ball straight up, describe the time-height relationship (as best you can) in mathematical terminology. (Do not use the words "up" or "down.")
- Does the ball reach a point where it doesn't go any higher?
- As time changes from 0 second to when the ball is at the high point, how is the height of the ball changing?
- As time changes from when the ball is at the high point to when it is on the ground, how is the height of the ball changing?
- How might you describe the height of the ball when it is between "increasing" in height and "decreasing" in height?
- What is the height of the ball when it is on the floor (relative to the CBR)?
- If you want to find a function that models the data from above, how would you select a suitable candidate?
- Find a model of the data collected above.
- When is the model "increasing?"
- When is the model "decreasing?"
- When does the model have a maximum value?
- When does the model have a value of zero?
- Given the function $Y_2(x) = -4.7(x-.16)^2 + 5.1(x-.16) + 0.4$, describe when it is increasing, decreasing, what the maximum value is, and find the zeros.

You are now ready to use the more traditional approach to the study of the behaviors of maximum, increasing, decreasing, zeros.