NEW EMPHASIS OF FUNDAMENTAL ALGEBRAIC COMPETENCE AND ITS INFLUENCE IN EXAM SITUATION

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Abstract
While in traditional mathematics education manual calculating skills are dominant we can now observe fundamental changes in the significance of certain algebraic competences. Due to the fact that calculating is done by the CAS we see a shift from doing to planing, a growing importance of other algebraic competences like finding of terms, structure recognition, testing, visualizing a.s.o. One of the most important results of the Austrian CAS-projects is the more pupil oriented and more experimental learning process when using CAS. Therefore we have to ask ourselves whether the traditional instruments in the exam situation are still valid and suitable to these new ways of learning.

1. The task of schools in the age of information technology and the role of mathematics

Education versus Qualification

Thesis1: The school of tomorrow in an effort to prepare pupils for life long learning must be able to do both: educate our youth and equip them with the necessary qualifications

Therefore, educators must not limit themselves to simply providing subject competence. In order to fulfill the mission of education it is necessary for teachers to equip the students with various key qualifications:

- subject competence - methodological competence
- social competence - personal competence

Ability versus competence

Another point of view which is important for the ideas of my lecture comes from a discussion with B. Kutzler about the difference between the concepts of „competence“ and „ability“

- The ability to do something is the quality or skill in doing a particular thing
- Having the competence includes more than just being able to do something. It means doing something with understanding and doing it based on a personal decision and because of one´s own personal considerations. Furthermore, competence implies that something is done well.

Thesis 2: In the school of tomorrow students should not only acquire abilities, they should gain personal competence

Short term – versus long term competence

Discussing pupils ability and competence we often watch the following common mistake:

There is no difference made in:

- The short term competence of having skills and knowledge readily available for a certain learning process and
- The long term competence of being able to recall and retrieve knowledge or ability much later to solve an actual problem

Thesis 3: The short term competence, which should especially be available for a certain learning process, must be more detailed and more extensive than the long term competence of recalling and retrieving.

2. The necessary fundamental algebraic competence in the age of CAS

We first have to ask ourselves what the educational value and the goals of the subject mathematics are and then consider the question of partial algebraic competence. I have based my thoughts concerning the grander picture of mathematics on the following definition by Bruno Buchberger

Mathematics is the technique, refined throughout the centuries, of problem solving by reasoning

Let us now turn to those competences which, in my opinion, are important:

2.1. The competence of finding terms or formulas

Of the three phases of the problem solving process, modelling—operating—interpreting, the operating phase has always dominated. The tools of CAS now make it possible to more evenly distribute the importance of the three phases. This means that developing formulas gains more importance in comparison to calculating with terms.

The influence of CAS:

- The CAS allows the students to transform a condensed "word formula" directly into a symbolic object of the mathematical language by defining variables, terms or functions or writing programs.
- The CAS allows a greater variety of prototypes of a formula and also offers some which were not available before. "
- The CAS offers and allows a greater variety of testing strategies, in this case testing if the formula is suitable for the problem and mathematically correct.

2.2. The competence of recognizing structures and recognizing equivalence of terms

This competence is necessary when developing a term, when deciding upon or entering a certain operation and also when interpreting or testing. This competence has always been of great importance as research, such as that of Günter Malle, has shown us that the most commonly made mistakes during algebraic operating are those of recognizing structures. But recognizing structures got a new meaning since tools like CAS are available.

The influence of CAS:

- When using a CAS the first step, the input of an expression, needs a structure recognition activity.
- Using the CAS as a black box for calculating a recognition of the structure of the expression is necessary before entering the suitable command.
- The learner must interpret results and recognize their structure which he himself did not produce.
- The individual results of various students doing experimental learning must often be checked for their equivalence.
- CAS sometimes produce unexpected results and students do not know whether they are equivalent to their expected results.

2.4. The competence of testing

A central result of our CAS projects is a more experimental and independent learning process, whereby the expert is not so much the teacher as the CAS. This means that testing becomes even more in important. The stronger emphasis on modelling and interpreting also demands a higher competence in testing.

The influence of CAS:

- The CAS enables the learner to carry out tests both more effectively and quickly.
- Completely new possibilities are available as far as algebraic and graphic testing are concerned
- Using CAS causes a new problem: The learner has to examine and to interpret results which he himself did not produce. The expectation of the sort of the solution or the form of the algebraic term sometimes differs between the learner and the machine.

- The variety of paths leading to solutions and therefore the number of different results increase dramatically. One will not often find the "algorithmic obedience" of the classical math classroom, in which the majority of the students simply imitate the strategies presented by the teacher. Therefore the equivalence of the numerous results has to be tested.
- The more applied mathematics which we see in the CAS-classrooms demands more testing of the correctness of the model, testing of the usefulness of the mathematical solution according to the given problem and testing of the influence of parameters.

2.4. The competence of calculating

Before discussing what sort and what extent of competence is necessary for the students to have we might first define what calculation competence is:

Definition: Calculation competence is the ability of a human being to apply a given calculus in a concrete situation purposefully.

This definition shows that calculation competence does not only mean to execute a certain operation. Most important for us is the distinction between the goals "perform an operation" (to some extent this can be delegated to a calculator) and "choose a strategy" (this cannot be done by the calculator.)

Manual calculating skills are a branch of the calculation competence, because having calculation competence could also mean being able to decide on the suitable algorithm and to delegate the execution to the computer. But we are still dedicated of the following thesis

Thesis 4: **For mathematics to develop within a learner certain calculation skills are needed.**

We teach manual calculation skills not only for their own sake, I am convinced that they are prerequisites for the attainment of most of the fundamental algebraic competences which I speak about in this lecture.

Another point of view:

Richard Skemp [Skemp, 1976] distinguishes between relational understanding and instrumental understanding (or shortly between understanding and skills):

Instrumental understanding: Mathematical usage of rules when solving problems without necessarily knowing why the rule is valid.

Relational understanding: The ability of deriving rules, interpreting and possibly proving, to see them as rules in a net of concepts ("knowing both, how to do and why).

This point of view leads to some questions:

Question 1: Is instrumental understanding a prerequisite or a support for a higher level of relational understanding?

Question 2: Does relational understanding support the necessary skills of instrumental understanding?

An investigation of our last Austrian CAS project gave us the following answers [Heugl, Gösing 1999]:

Answer to question 1: Instrumental understanding is not an absolutely necessary prerequisite for a higher relational understanding. The ability to give reasons does not automatically go hand in hand with the ability to calculate.

Answer to question 2: Relational understanding does not necessary support the skills of instrumental understanding. Or in other words: To have relational understanding is not enough for having instrumental skills.

The influence of CAS in the calculation competence:

- A shift in emphasis from calculating skills to more conceptual understanding, to more modelling and interpreting.
- A shift from doing to planning.
- A reduction of the complexity of manual calculated expressions.
- A shift from calculation competence to other algebraic competences, like structure recognition competence or testing competence.
- A better connection between the formal aspect of mathematics and the aspect of contents.

2.6. The competence of visualizing

A special quality of mathematics is the possibility of graphic representation of abstract facts. Visualizing was also important in traditional mathematics education but it was not easy to get the graphic prototype of a concept or a function. Apart from free hand drawings, it is difficult to develop graphs without using a computer.

The influence of CAS:

- CAS allows the learner to get the graph faster and more directly.
- Other prototypes of a concept or specially a function are also available much more easily, like a table or lists of values or matrices in a Data/Matrix editor.
- The CAS allows the learner to use several prototypes parallely, while in traditional math education only one prototype was given.
- The learning process consists of shuttling between several prototypes that means shuttling between several windows. Therefore we call this didactical concept the Window Shuttle Method.
- These facts also allows to solve algebraic problems graphically.

2.6. The competence of working with modules

Using modules is not new for the learners. But the computer, and especially CAS, opens a new dimension of modular thinking and working.

The influence of CAS:

- By defining or storing parts of a complex expression as a variable, students can simplify the structure of the expression making it more comprehensible and they can calculate with such modules.
- Students now more often create new language elements by using the name of the expressions instead of the expressions themselves.

2.7. The competence of using the chosen CAS

The influence of CAS:

- The use of CAS causes additional demands and problems for the students. The operation of the electronic tool needs additional skills which also have to be practiced as calculation skills.
- The evaluation of our last project shows that the measured growing joy and interest in mathematics is significantly higher by those pupils who have no problems with the operation of the computer.
- The necessary commands, operations and modes have to be offered to the students in small portions. Practicing and repeating in regular intervals are necessary.
- The use of CAS as a Black Box for problem solving demands an agreed documentation of the way of solution, especially in the exam situation.

3. The influence of the use of the tool CAS in the exam situation

In the past the exam situation has always had a great influence on the content and the didactic concept of mathematics education. So the

emphasis sometimes placed on a specific math topic can only be explained because it is easy to construct a suitable test.

Already in our former CAS projects we recognized that the way of assessment was not suitable to the new ways of learning which we observed in our CAS classes.

In traditional mathematics education written exams (5 or 6 one-hour-tests per year) dominate. As far as content is concerned the emphasis is on calculation skills. This way of testing is suitable to the dominating style of teacher centered teaching, which causes a more reproductive way of learning.

Some new ways in the exam situation

Based on this recognition we started our recent project. Task was to investigate the consequences of the following models of examining the students learning:

***Model 1:* "A year´s-time for written exams"**

In contrast to the Austrian National Curriculum which prescribes a certain number of hourly written exams per year, our model allows the teachers to test a total of 250 minutes in the academic year, thus permitting them to assess their students in two different ways:

- Shorter tests (15 to 30 minutes) to examine certain fundamental competence like calculation competence, visualization competence or also abilities of using the available CAS. Mostly the use of the tool CAS is not allowed.
- Problem-solving-examinations (50 to 120 minutes) measure the competence of problem solving with more application-oriented examples, with more open questions, with more emphasis on argumentation, reasoning or interpreting. During these examinations

students mostly are allowed to use their learning media like their math school books or their exercise books.

Both the model and the content of the tests were influenced by our discussion about fundamental mathematical competence:

Thesis 5: The fundamental competence examined by short tests is the basis and the prerequisite for the most important task of mathematics, the solving of problems.

Significant for this model is the idea of the two phases:

- At first building the foundation by focussing on a certain mathematical fundamental competence like an algebraic competence and then
- in a second phase using several fundamental competence for problem solving.

I would like to stress my statement that in a learning phase which is focussed on algebraic calculation skills I expect higher abilities than those we demanded as a "long-term competence" in our paper.

***Model 2:* "Project work"**

A certain number of the classic written exams are substituted by projects which are partly done during the lessons but the larger part of the work the students have to do at home.

The advantage of this model of examination is that the learning process and the phase of assessment are not separated. Examination is not a singular event which often causes a lot of stress by the students and the result of which often depends on the momentary state of mind of the students. Anyway this model encourages not only the mathematical competence, it also strengthens the other key qualifications, like methodological competence, social competence and personal competence.

Another advantage is that this model allows an inner differentiation which is not possible when using common written tests: More gifted students can work more on demanding problems than not so gifted students

Model 3: **Cross curriculum tests**

One of the main tasks of the school of the future is a greater emphasis on training of networked-thinking. In comparison with traditional classes in our CAS-classes we watch a growing importance of cross curriculum phases and therefore it was reasonable to consider this fact in the exam situation.

References

- Herget, W. [1995]: Save the idea – save the recipes! In: Rechenfertigkeit und Begriffsbildung; Tagungsband der 13. Arbeitstagung des Arbeitskreises "Mathematik und Informatik" der GDM in Wolfenbüttel; Sept. 1995, p 156.
- Heugl, H. [1999]: The necessary fundamental algebraic competence in the age of CAS Lecture at the 5th ACDCA conference in Gösing, Austria, August 1999. ACDCA Home page: http://www.acdca.ac.at
- Skemp, R. [1976]: Relational Understanding and Instrumental Understanding. In: Mathematics Teaching , 77 (1976), 16-20.