

Chicken Fence Problem

Life ain't easy for Mr. Fox: his wife pesters him all the time to make a new fence for chicken. Since the old one has been demolished by the storm the chicken walk happily around the garden leaving their »output« on the terrace and on the garden furniture. On this Sunday morning Mrs. Fox puts her foot down: » Get the chicken behind the fence now or you shall spend the night in the garden!« So Mr. Fox wanders to the shed where he finds 10 m of wire and some sticks.

The chicken fence will be rectangular. One side of the rectangle will be on the wall, the perimeter of the remaining three sides will be 10 m. As an animal friend Mr. Fox wants to give his beloved chicken as much space as possible. Which dimensions of the rectangle will give the maximal area?

We will try to solve the problem in three ways: using table(spreadsheet), graphically and analitically (without using derivative*), using GeoGebra .

1. Visualisation of the Problem

The applet shows the rectangle limited by the wall on the left. By moving the slider, the dimensions of the rectangle change.

2. Table

We use the point C (base, height) and the point S(base, area) . By moving the slider the coordinates of C and S fill the table .

We can read the base and the height giving maximal area from the table.

3. Graph

The graph will show area as a function of base. It can be obtained in two ways: either directly by moving the slider (in this case the table should be disabled) or by plotting the points that correspond to the table (as we normally do in the classroom).

Reading the coordinates of point S at its the highest position we get the result .

4. Analytical Solution

Area can be expressed as a quadratic function of the base (x) under the condition :
 $x + 2h = 10$, where h is the height of the rectangle.

By calculating the coordinates of the vertex we get the result. It can be proved by writing the quadratic equation into GeoGebra's Input Bar to get the graph. If our equation is correct, the points from section 3 will lie on it. By using the command *Extreme* the coordinates of vertex will appear in the algebraical window.

Possible discussion: Are all three results equal? Why the differences? Which method is the best? Which is the fastest?

Dilemma of Mr. Fox

Just before starting the work Mr. Fox gets confused by a new idea. Maybe the chicken would prefer a fence in the shape of isosceles triangle with the base on the wall. (x – base, $2a = 10$). Find out the maximal area of the triangle and compare with the one of rectangular shape!

Finally, when the fence is made and the chicken are inside, Mrs. Fox still finds reasons for criticism.

» It's not big enough! It will be too small in just a few days! Besides, the rectangular shape is totally out for a hen-house! Just look at the neighbour's! That's really something!«

Is it possible to build a larger fence under starting conditions by avoiding edges? For example: half of a circle ($2r = 10\text{m}$) or quarter of a circle ($r + \text{arc} = 10\text{ m}$)? Have you got other ideas?

