

## Paper Box Problem

**A rectangular piece of cardboard (40 cm x 50 cm) is used to make a box without the lid in the following way: squares with the side  $x$  are cut out of each corner. The rest is folded up and stuck together to form a box. Find the value of  $x$  to get maximal volume.**

We will try to solve the problem in three ways: using table(spreadsheet), graphically and analitically (without using derivative\* ), using GeoGebra as a tool.

### 1. Visualisation of the Problem

The applet shows the cardboard and the slider at initial position. By moving the slider the box appears changing its shape. We try to guess the value of  $x$  where the volume is maximal and maximal volume of the box.

### 2. Table

The slider is at initial position, the point  $Vol(x, y)$  at origin. By moving the slider the point  $Vol(x, y)$  moves in the coordinate system, its coordinates filling the table .

From the table we can read the value of  $x$  and maximal  $V(x)$  (the values are approximate). It will be interesting to compare the result to our guess in section 1. (Many students are really week at estimating results.)

### 3. Graph

The graph can be obtained in two ways: either directly by moving the slider (in this case the table should be disabled) or by plotting the points that correspond to the table (as we normally do in the classroom).

Reading the coordinates of the highest point we get  $x$  and maximal  $V(x)$  .

### 4. (Partly) Analytical Solution

$V(x)$  can be expressed as a polynomial of 3. degree. Writing its equation into GeoGebra`s Input Bar its graph will appear. If our equation is correct, the points from section 3 will lie on it. By using the command Extreme the coordinates of both extrema will appear in the algebraical window.

#### Discussion:

- Why is the graph from section 3 different from the graph from section 4?
- What about minimum?
- Are the results in section 2, 3 and 4 equal? If not, which are the most accurate? How to improve the accuracy?

\*The derivation is taught in the middle of grade 4. Because of time limitations we were not able to solve the problem in the class »properly« using derivation as it will be done in near future.